Derivatives

(IV) Black Scholes Model [BSM]

$$C_0 = S_0 \times N(D_1) - \frac{K}{e^{rn}} \times N(D_2)$$

where,

$$D_1 = \frac{\ln\left[\frac{S_0}{K}\right] + \left[r + \frac{\sigma^2}{2}\right]t}{\sigma\sqrt{t}}$$

$$D_2 = D_1 - \sigma\sqrt{t}$$

$N(D_1)$, $N(D_2)$ = Probability using PDC of D_1, D_2.

Q.35 (i) $S_0 = 415$

$n = 3/12$

$K = 400$

$r = 5\%$

$\sigma = 22\%$

$$C_0 = S_0 \times N(D_1) - \frac{K}{e^{rn}} \times N(D_2)$$

$$= 415 \times N(D_1) - \frac{400}{e^{0.05 \times 3/12}} \times N(D_2)$$

$$= 415 \times N(D_1) - \frac{400}{e^{0.0125}} \times N(D_2)$$

$$= 415 \times N(D_1) - \frac{400}{1.0126} \times N(D_2)$$

$$= \boxed{415 \times N(D_1) - 395.02 \times N(D_2)}$$

$$= 415 \times 0.69346 - 395.02 \times 0.653772$$

$$= 29.53$$

$$D_1 = \frac{\ln\left[\frac{S_0}{K}\right] + \left[r + \frac{\sigma^2}{2}\right]t}{\sigma\sqrt{t}}$$

$$= \frac{\ln\left[\frac{415}{400}\right] + \left[0.05 + \frac{0.22^2}{2}\right]3/12}{0.22 \times \sqrt{3/12}}$$

Read Shortcut Calci steps at end of qstn for $\ln\left[\frac{415}{400}\right]$

$$= \frac{\ln[1.0375] + 0.01855}{0.11}$$

$$= \frac{\frac{\log 1.0375}{\log e} + 0.01855}{0.11}$$

$$= \frac{\frac{0.0161}{0.4343} + 0.01855}{0.11}$$

$$= 0.5056$$

$$D_2 = D_1 - \sigma\sqrt{t}$$
$$= 0.5056 - 0.11$$
$$= 0.3956$$

How to calculate log 1.0375

(i) How many digits before decimal? 1

(ii) ∴ Characteristic = 1 − 1 = 0

Fixed

Final ans. will be 0. *.

(f) (iii) Find Mantessa using log Tables
While checking Tables, ignore Decimal i.e.

$$\begin{array}{ccc} & 1038 & \\ \swarrow & \downarrow & \searrow \\ \text{Row} & \text{column} & \text{mean Difference} \\ & 0128 & + \quad 33 \\ & 0161 & \end{array}$$

$\log 1.0375 = 0.0161$

How to calculate log e i.e. log 2.71828

0.4343
↓
By Heart this number

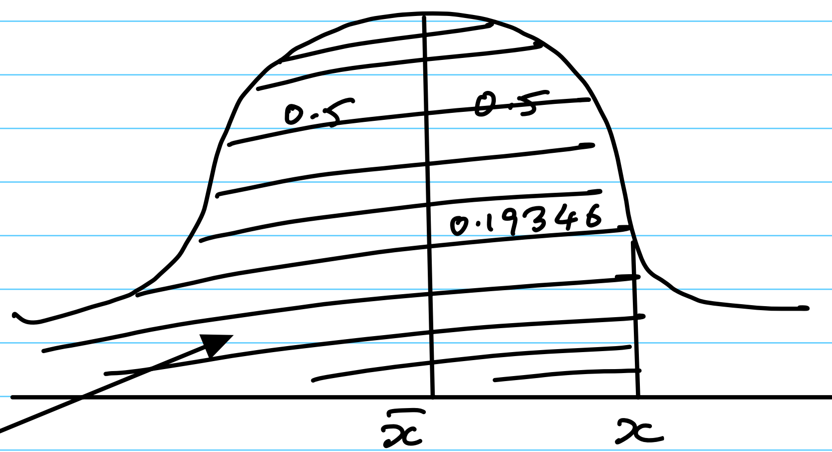
$\therefore \ln[x] = \dfrac{\log x}{0.4343}$ or $\log x \times 2.3026$

Calculation of N(D₁)

$D_1 = 0.5056$ → Consider this as z
[x, $\bar{x}$ & σ_x are not known & need not be known]

↓

What to consider as shaded area?



Shaded Area is Entire Area to the LEFT of x.

we have placed x to the Right of $\bar{x}$ ∵ D_1 [z] is (+); $z = \frac{x - \bar{x}}{\sigma_x}$ → for this number to be (+), $x > \bar{x}$

Cal. of $N(D_1)$

$P[0.5056] = ?$

$P[0.50] = 0.1915$
$P[0.51] = 0.1950$

0.50	0.5056	0.51
0.1915	?	0.1950

$z \uparrow$	$P \uparrow$
$0.01 \uparrow$	$0.0035 \uparrow$
$[0.51 - 0.50]$	$[0.1950 - 0.1915]$
$0.0056 \uparrow$	$0.00196 \uparrow$
$[0.5056 - 0.50]$	

$\therefore P[0.5056] = 0.1915 + 0.00196$
$= 0.19346$

$\therefore N(D_1) = 0.5 + 0.19346$
$= 0.69346$

Alternate Method of Interpolation :

$P[0.5056] = ?$

$P[0.50] = 0.1915 + 0.5 = 0.6915$

$P[0.51] = 0.1950 + 0.5 = 0.6950$

0.50	0.5056	0.51
0.6915	?	0.6950

$z \uparrow$	$P \uparrow$
$0.01 \uparrow$	$0.0035 \uparrow$
$[0.51 - 0.50]$	$[0.6950 - 0.6915]$
$0.0056 \uparrow$	$0.00196 \uparrow$
$[0.5056 - 0.50]$	

$\therefore P[0.5056] = 0.6915 + 0.00196$
$= 0.69346$

$\therefore N(0, 1)$

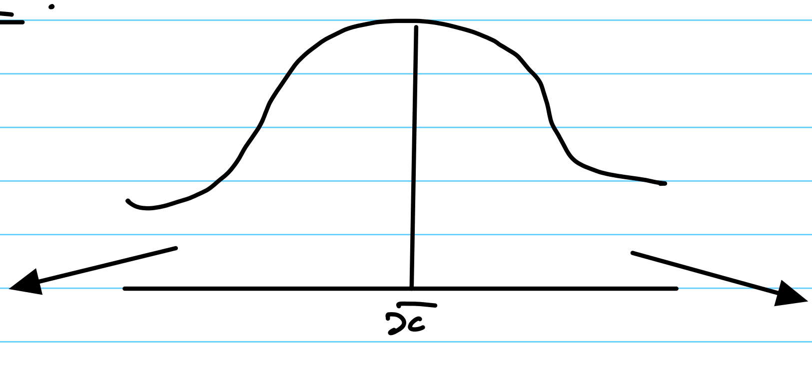
Alternate Method of Interpolation :

[Check Z value table on Pg 342]

$P[0.5056] = ?$

$P[0.50] = 0.6915$
$P[0.51] = 0.6950$
[Now same as above]

GK :



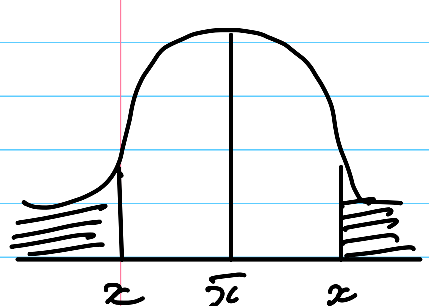
Two Tailed Hypothesis

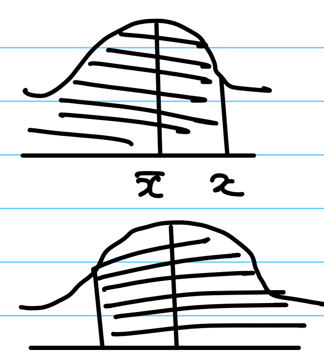
Value of One Tail / Probability of One Tail

↓

If given in Qstn

< 0.5 | > 0.5





x $\bar{x}$

Calculation of N(D₂)

$P(0.3956) = ?$

$P(0.39) = 0.1517$
$P(0.40) = 0.1554$

0.39	0.3956	0.40
0.1517	?	0.1554

Z ↑   P

0.01 ↑   [0.40 − 0.39]   0.0037 ↑   [0.1554 − 0.1517]

0.0056 ↑   [0.3956 − 0.39]   0.002072 ↑

$\therefore P[0.3956] = 0.1517 + 0.002072$
$= 0.153772$

$\therefore N(D_2) = 0.153772 + 0.5$
$= 0.653772$

Calci Steps for $\ln\left[\frac{415}{400}\right]$

1. $\frac{415}{400}$
2. Press $\sqrt{\ }$ 12 Times
3. − 1
4. Multiply by 4096

$= 0.03681$ → ICAI approved!

(ii) $C_0 = 380 \times N(D_1) - 395.02 \times N(D_2)$

$D_1 = -0.2977$

$D_2 = -0.4077$

Cal. of $N(D_1)$

$P[0.2977] = ?$

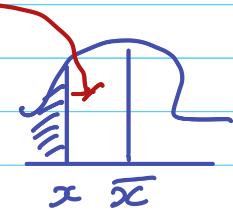
$P[0.29] = 0.1141$

$P[0.30] = 0.1179$

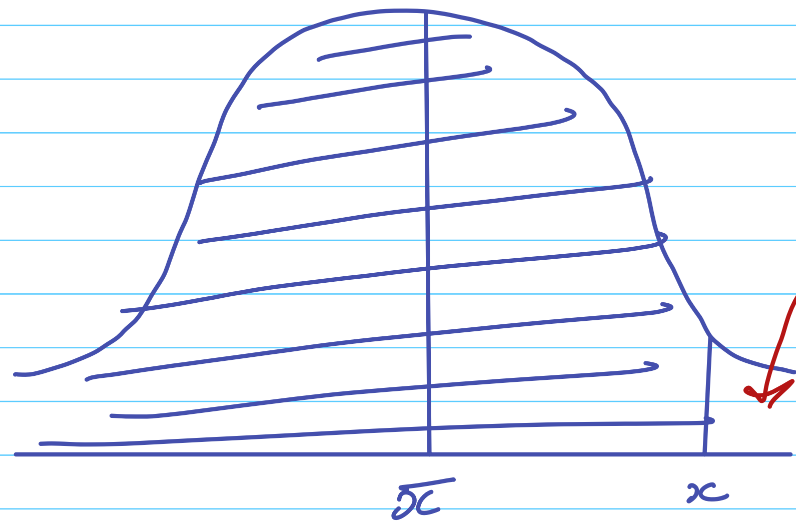
0.29	0.2977	0.30
0.1141	?	0.1179

$z \uparrow$	$P \uparrow$
$0.01 \uparrow$ $[0.30 - 0.29]$	$0.0038 \uparrow$ $[0.1179 - 0.1141]$
$0.0077 \uparrow$ $[0.2977 - 0.29]$	$0.002926 \uparrow$

$\therefore, P[0.2977] = 0.1141 + 0.002926$
$= 0.117026$



$\therefore, N(D_1) = 0.5 - 0.117026$
$= 0.38297$



$P[0.5817] = ?$

$P[0.55] = 0.2912$

$P[0.60] = 0.2742$

0.55	0.5817	0.60
0.2912	?	0.2742

$z \uparrow$	$P \downarrow$
$0.05\uparrow$ $[0.60 - 0.55]$	$0.017\downarrow$ $[0.2742 - 0.2912]$
$0.0317\uparrow$ $[0.5817 - 0.55]$	$0.010778\downarrow$

$\therefore P[0.5817] = 0.2912 - 0.010778$
$= 0.2804$

$\therefore N(d_2) = 1 - 0.2804$
$= 0.7196$